Following the golden rule: Negishi welfare weights without apology

By Joshua K. Abbott[1*] and Eli P. Fenichel[2]

[1]Arizona State University, Global Institute of Sustainability and School of Sustainability, PO Box 875502, Tempe, AZ 85287, joshua.k.abbott@asu.edu, 480.965.5528 (phone), 480.965.8087 (fax)

[2] Yale School of Forestry & Environmental Studies, 195 Prospect St., New Haven, CT 06511, eli.fenichel@yale.edu, 203.432.5114

* = Contact author

Authorship is equally shared.

ABSTRACT

It is often necessary to specify social welfare functions (SWFs) to allocate forms of capital that provide public services to heterogeneous agents yet lie outside of capital markets. Of the SWFs consistent with Pareto optimality, only weighted combinations of individual utilities with "Negishi", inverse marginal utility of the numeraire (income), weights replicate market outcomes for given initial resource allocations. This feature motivates Negishi weights' extensive use in applied economics, including in high-profile integrated assessment models of economic growth and climate change. Nevertheless, Negishi weights are often criticized on ethical grounds. We show the Negishi SWF is ethically consistent with the 'Golden Rule,' whereby the social planner weights others' utilities at a point in time as he would weight his own future discounted utility. This result clarifies the implicit ethical assumptions embodied in many economic models. We also show that a specific link exists between Negishi weights and a generalization of the Ramsey discounting rule.

Keywords: social welfare function, golden rule, distribution, discounting

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1. Introduction

Many major challenges facing society today (climate change, emerging infectious disease, deforestation) relate to allocating public capital stocks through time and across peoples. Economically efficient investment in and use of public capital stocks (infrastructure, greenhouse gas abatement, air quality, biodiversity, fish stocks, and forests) requires that the time path of decisions that influence these stocks maximize the present discounted value of welfare. However, the institutional preconditions for such allocations to occur in a market economy, such as excludability of property rights and complete spot and futures markets rarely exist for these resources, opening a potential role for public policy in mitigating the associated dynamic and static externalities.

A capital theoretic approach to policy focuses on constrained dynamic optimization of an objective function as an intertemporal measure of welfare, often expressed as a current value Hamiltonian (Weitzman 1976; Brock and Xepapadeas 2003; Partha Dasgupta and Maler 2000). However, the definition of 'welfare' in this context rests on the choice of the objective functional that is subject to maximization, where the choice of this criterion must be economically meaningful in order to have welfare-theoretic justification.

In seeking a dynamically efficient policy, a decision maker faces the dilemma that the set of relevant agents, be they individuals, countries, or coalitions (North vs. South), may be quite heterogeneous with respect to their endowments and preferences. The process of finding an optimal policy is inextricably linked with the question of how to resolve this heterogeneity into a coherent single measure of welfare by defining the rates of substitution between heterogeneous agents' welfare. This problem inherently involves making inter-agent comparisons of welfare (interpersonal comparisons of utility), a controversial practice in economics. A direct approach to the challenge of resolving this heterogeneity into a coherent single measure is to maximize an explicit social welfare function (SWF) (Bergson 1938; Samuelson 1956). There are innumerable potential social welfare functions, each consistent with distinct ethical positions vis-à-vis the distribution of welfare across individuals (Mueller 2003). However, if the maxima from a SWF are constrained to follow the (semi-strong) Pareto principle then they must (up to an affine transformation) consist of a weighted sum of individual utility functions (Harsanyi 1955; Domotor 1979; Mandler 2005), where the weights on individual utilities are positive (non-negative).[1]

Within this class of Pareto-consistent SWF, one weighting strategy has seen disproportionate use. In 1960 Takashi Negishi demonstrated how any static market equilibrium can be characterized as emerging from the maximization of a linear combination of individual utilities, where the weights are equal to the inverse of the marginal utility of the numeraire good (income) for each individual evaluated at the maximizing (equilibrium) resource allocation (Negishi 1960). In addition to providing a way to work backwards from a particular market equilibrium to its inherent SWF, Negishi's proof provides an iterative forward algorithm to start from initial endowments to simulate a static market equilibrium consistent with a set of initial endowments (Negishi 1972). First, the SWF is maximized with an arbitrary guess at the weights. Second, consistency of the suggested allocations with individual budget constraints is checked. Third, given over- or under-expenditure by different agents, the weights are adjusted to better align income with expenditures. Fourth, the process is iterated to convergence.

[1] The (semi-strong) 'Pareto principal' implies that the social welfare function (weakly) increases if a change in the allocation of resources is (weakly) preferred by all agents included in the SWF (Weymark 1993).

Upon convergence, the marginal rates of substitution between goods define relative prices in equilibrium. Negishi weights are 'market replicating' since they provide a mechanism for simulating a Pareto efficient market allocation that is consistent with an initial distribution of income. This property naturally extends to the efficient pricing of externalities that would otherwise lie outside the market. Furthermore, the Negishi weighting approach can be extended to intertemporal general equilibrium models to simulate the outcome of an efficient dynamic market (Kehoe 1989). Negishi weights have played a substantial computational role in computable general equilibrium (CGE) models with multiple regions and trade (Ginsburgh and Keyzer 1997). Perhaps the most prominent CGE application of dynamic Negishi weights has occurred in regionally disaggregated integrated assessment models such as RICE (William D. Nordhaus and Boyer 2000; William D Nordhaus and Yang 1996; Yang and Nordhaus 2006) and MERGE (Manne et al. 1995; Manne and Richels 1995) where the weighting serves to simulate the outcome of an efficient international market for carbon dioxide (CO_2) or CO_2-equivalent emissions.

Negishi weights have a unique property in that they are the only weights (up to a constant of proportionality) that preserve the initial distribution of income – allowing exchange but not redistribution. Maximization of a purely utilitarian (equally weighted agents) SWF tends to equalize the marginal utility of consumption across agents by redistributing resources from relatively wealthy to relatively poor agents. By dividing each agents' utility by their marginal utility of income at the optimum, a value that is smaller for wealthier agents than for otherwise identical poorer agents, Negishi weights cause the marginal utility of consumption for relatively wealthy agents to be less at the SWF maximum than for relatively poor agents. The resulting weights can be viewed as converting each agents' utility function into money-metric (numeraire-metric) units according to each agent's own valuation of income. These rescaled utilities are then summed across agents so that Negishi weights are 'market egalitarian' in the sense that every agent's dollar is weighted equally at the optimum. However, this is achieved by placing disproportionate weight on the utility of relatively wealthy agents. In the context of climate integrated assessment models, Negishi weights have been defended as describing the outcome in a scenario in which missing spot and futures markets for carbon are completed along an optimal abatement strategy, conditional on the current global distribution of wealth (Yang and Nordhaus 2006). All agents face a 'law of one price' for carbon abatement along this trajectory.

Negishi weights have come under attack for justifying the preservation of existing inequalities in policy contexts for which concerns of efficiency and distribution are not easily negotiated separately (Stanton 2011; Tol 2001). A related criticism lies in the often cursory and technical manner in which Negishi weighting is introduced and motivated in models for policy analysis – if, indeed, it is discussed at all. When explicitly mentioned, Negishi weights are often justified as a way to separate questions of efficient resource allocation from concerns of uneven development (Keller et al. 2003) or simply as a way to yield familiar Pigouvian solutions to externality problems (Fullerton and West 2002).

This rhetorical treatment belies the pivotal role this particular weighting of welfares has in shaping model outcomes and implied policy prescriptions. Perhaps, a desire to avoid this discussion is the reason Negishi SWFs are seldom discussed. Nevertheless, aggregating individual welfare after weighting by the inverse marginal utility of income is ubiquitous. The summation of money-denominated and discounted benefits and costs across heterogeneous agents to form a single measure of net benefits of a policy dominates the state of practice in benefit-cost analysis, but the normative justification of this measure of net benefits in Kaldor-

Hicks potential compensation implicitly rests on the 'market egalitarian' weighting of the Negishi SWF (Hammitt 2013). While acknowledging the formidable ethical choices embodied in any SWF, this paper offers a simple, intuitive proof that Negishi weights in a dynamic setting do possess a desirable ethical property. They satisfy a version of 'The Golden Rule': Do unto others as you would do unto yourself. We demonstrate this general principle using a standard model of a single dynamically optimizing agent with sole ownership over a depletable capital good. This analysis can serve as an intuitive justification for what is likely the most commonly specified or implicit SWF in economic analysis.

2. The 'Golden Rule'

We begin by considering the case of a sole owner of a capital stock, *s*, with concave preferences $U(c(t), z(t), s(t); \xi(t))$ defined over the capital stock as well as some form of extractive consumption, *c*, and a numeraire good, *z*. ξ is a preference parameter that is allowed to change exogenously over time – accounting for potential changes in individual preferences. We include the capital stock in the utility function to consider cases in which benefits from the capital stock are conferred from 'non-market services' (amenity effects of climate) as well as extractive consumption. We assume that the individual receives a predetermined, exogenous flow of income, $y(t)$ and that the price of extractive consumption is p, so that $z(t) = y(t) - pc(t)$.[2] Consumption degrades the capital stock according to the equation of motion $\dot{s} = G(s, c)$ where $G_c < 0$.[3]

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[2] Extending the model to endogenize income in the manner of optimal growth models is transparent and adds nothing to the insights of our model.

[3] This equation of motion can encompass traditional physical capital accumulation and depreciation, nonrenewable resources (i.e. minerals), renewable resources (i.e. fisheries and forests) and stock pollutants (i.e. CO_2).

We assume that the sole owner maximizes the net present value of his utility by choosing a consumption path subject to the equation of motion of the capital stock. We assume exponential discounting of utility at a constant rate to ensure time consistency of the optimal consumption path (Strotz 1956). We also assume, without loss of generality, that the maximization begins at some arbitrary point in time $t = \theta$ and extends indefinitely into the future.

$$(1) \qquad \max_{c(t)} \int_{\theta}^{\infty} U(c(t), y(t) - pc(t), s(t); \xi(t)) e^{-\rho(t-\theta)}\, dt$$
$$s.t. \quad \dot{s} = G(s(t), c(t)), \; s(\theta) = s_{\theta}$$

It is well known that a constant rate of utility discounting prescribes a time-varying rate of discount for consumption, the numeraire good, or any other time-varying control or state variable inside the utility function (Arrow and Kurz 1970; Partha Dasgupta and Heal 1979). One such rate is the numeraire discount rate.

Definition 1: The <u>numeraire discount rate</u>, $r(t)$, is the rate that the present value of the marginal utility of the numeraire diminishes over time along a given path of $c(t), y(t), and\ s(t)$: $r(t) = -\frac{1}{U_z e^{-\rho t}} \frac{dU_z e^{-\rho t}}{dt}$.

Given predefined paths of consumption, income and the stock of natural capital, the numeraire discount rate measures the rate of change in the utility valuation of an infinitesimal bit of the numeraire as it is shifted forward in time. It is straightforward to show in this case (Heal 1998) that

$$(2) \qquad r(t) = \rho + \eta_{zz}\frac{\dot{z}}{z} + \eta_{zc}\frac{\dot{c}}{c} + \eta_{zs}\frac{\dot{s}}{s} + \eta_{z\xi}\frac{\dot{\xi}}{\xi},$$

where $\eta_{zz}, \eta_{zc}, \eta_{zs},$ and $\eta_{z\xi}$ refer to the own and cross-elasticities (all multiplied by -1) of the marginal utility of the numeraire. This implies that the numeraire discount rate is composed of the utility discount rate plus terms that reflect the growth rate of extractive and numeraire consumption, the capital stock, and exogenous variation in preference parameters multiplied by terms that reflect the sensitivity of the valuation of the numeraire to these changes. In the event that the utility function is separable in the numeraire and preference parameters are stable this term reduces to $r(t) = \rho + \eta_{zz}\frac{\dot{z}}{z}$, which is the "Ramsey" form (Ramsey 1928) commonly seen in simple growth models without the capital stock entering utility. This indicates that the numeraire discount rate will tend to exceed the utility discount rate in times of expanded numeraire consumption, reflecting diminishing marginal utility of the numeraire.

With the previous definition in hand, we can define the Negishi-weighted optimization

$$(3) \qquad \max_{c(t)} \int_{\theta}^{\infty} e^{-\int_{\theta}^{t} r^*(\tau)d\tau} U_z^*(t)^{-1} U(c(t), y(t) - pc(t), s(t); \xi(t))\, dt$$
$$s.t. \quad \dot{s} = G(s(t), c(t)), \;\; s(\theta) = s_\theta$$

$U_z^*(t)^{-1}$ is the inverse marginal utility of the numeraire, and therefore the marginal utility of income, prescribed under the maximization program in Eq. (1). The inverse marginal utility of the numeraire is solely a function of time. Whereas Eq. (1) described the maximization of the present value of utility through time, Eq. (3) describes the maximization of a weighted stream of utilities through time, where the weighting decreases in the marginal utility of the numeraire. Furthermore the weighting places utility in 'money metric' units of the numeraire good. The form of the modified discount factor $e^{-\int_{\theta}^{t} r(\tau)d\tau}$ holds for the case of a generic, time-dependent discount rate (Caputo 2005) and the particular substitution of $r^*(t)$ leads to discounting with respect to the temporally varying value of the numeraire evaluated along the optimal path.

Proposition 1: The maximization of discounted un-weighted welfare through time for an individual can be equivalently expressed as the maximization of the Negishi-weighted and numeraire-discounted sum of utility through time.

The proof consists in showing that the integrands in Eqs. (3) and (1) are equivalent up to a multiplicative constant. First, we substitute the numeraire discount rate in for the discount factor term in Eq. (3).

(4) $$e^{-\int_{\theta}^{t} r^*(\tau)d\tau} = e^{\int_{\theta}^{t}\frac{1}{U_Z^* e^{-\rho\tau}}\frac{dU_Z^* e^{-\rho\tau}}{d\tau}d\tau} = e^{\int_{\theta}^{t}\frac{dln(U_Z^* e^{-\rho\tau})}{d\tau}d\tau} = e^{\ln\left(U_Z^*(t)e^{-\rho t}\right)-\ln\left(U_Z^*(\theta)e^{-\rho\theta}\right)} = \frac{U_Z^*(t)e^{-\rho t}}{U_Z^*(\theta)e^{-\rho\theta}} = \frac{U_Z^*(t)}{U_Z^*(\theta)}e^{-\rho(t-\theta)}$$

Upon substituting Eq. (4) into Eq. (3) the numerator of the discount factor in the integrand cancels the inverse marginal utility of income weights yielding

(3′) $$\max_{c(t)} \frac{1}{U_Z^*(\theta)}\int_{\theta}^{\infty} e^{-\rho(t-\theta)}U(c(t), y(t) - pc(t), s(t); \xi(t))\, dt$$
$$s.t. \quad \dot{s} = G\big(s(t), c(t)\big),\ s(\theta) = s_\theta$$

The objective function in (3′) is Eq. (1) rescaled by the optimal marginal utility of the numeraire at time θ. Therefore, the optimal control and state trajectories for Eq. (1) and (3) must be identical.

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The Negishi weighting dynamically normalizes the utility function at each instant so that the marginal utility of income for the rescaled utility is unitary throughout time. This dynamic rescaling transforms a general utility specification to be locally quasilinear so that the process of allocating the capital good across time to maximize the present discounted flow of utility is rendered equivalent at the margin to an agent engaging in intertemporal trade with different realizations of himself – with different endowments and even preferences – so as to maximize discounted total monetary surplus.[4] If the agent would prefer to consume more of his capital today in a myopic sense, but exercises forbearance along the optimal trajectory, then it must be the case that the present value of numeraire-discounted future surplus is sufficient in present

[4] Viewed from a complementary perspective, Negishi weights, if inverted, serve to rescale measurable ‘nominal’ income into ‘real’ utility units without distorting the nominal unit (Weitzman 2001).

value terms to fully compensate the agent for his self-control. Negishi weights combined with numeraire discounting create an analogy between fundamentally static ‘potential compensation’ measures of welfare adopted in benefit-cost analysis of policies with heterogeneous impacts across users and the process of internal arbitrage undertaken by a single dynamically optimizing agent. A discounting and utility maximizing agent prefers an adjustment of the consumption/investment path if the winner (the present or future self) can compensate the loser (again, the present or future self) for his sacrifice.

This correspondence between the Kaldor-Hicks justification of static benefit-cost analysis and the capital theoretic valuation embodied in discounted utility maximization relies critically on the *combination* of Negishi weights with their complementary time-varying numeraire discount rates. It is easily proven that only time-constant affine transformations of the paths of the numeraire discount rates or Negishi weights will maintain the equivalence between (3) and (1).[5] While Negishi weights rescale utility to be ‘money metric’ at the margin along the optimal path, the numeraire discount rate utilizes all available information on the rates of change of consumption, numeraire, capital stocks and time-varying preferences to correct for the time-varying rate of exchange between utility and numeraire.

The linkage between the Kaldor-Hicks justification of static benefit-cost analysis and the capital theoretic valuation embodied in discounted utility maximization is robust to the possibility that the agent’s endowments (y) and preferences (ξ) could vary exogenously over time with perfect foresight, which leads directly to a corollary.

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[5] However, discounted utility maximization can be viewed in terms of alternative definitions of the numeraire good. For example (3) can be recast with weights in terms of the inverse marginal utility of extractive consumption with the discount rate being changed to reflect the consumption rate of discount.

Corollary 1: The Golden Rule. Maximizing an additive social welfare function across heterogeneous agents using Negishi weights and numeraire discounting yields an outcome such that the procedure for weighting of individual utilities across the population at a point in time is identical to that employed by a single dynamically optimizing self-interested individual in weighting his own utility through time.

We have demonstrated that if an agent looks forward in time, to periods in which his income endowment or preference parameters change will, and maximizes the sum of discounted utility, then he will behave as if he is weighting his utility (before discounting) along the optimal trajectory according to the inverse marginal utility of income at each instant, where these weights will incorporate changes in tastes and endowments. The associated monetized objective function in Eq. (3) is, due to the linearity of discounted utility maximization, effectively a Pareto-consistent SWF with Negishi weights (and discounting) applied to future utility functions for a single individual. Therefore, the manner in which a single optimizing individual weights his current and future utility across two different states of preferences and endowments is, after accounting for the rate of time preference and temporal changes in endowments or preferences through the numeraire discount rate, exactly equivalent to the way that a social planner weights these same states at a point in time using Negishi weights.

These observations show that Negishi weights, despite the criticisms levied at them, possess some attractive ethical properties in addition to their pragmatic role in replicating dynamically complete market outcomes. In particular, they are ethically consistent in the sense that they apply the same weighting procedure across heterogeneous individuals that a self-interested individual would apply to himself across time. While they do preserve the existing distribution of income across agents, they nevertheless weight welfare across agents in a way that is consistent with the way that a single optimizing agent facing an exogenous time path of endowments would weight his own welfare through time. They therefore satisfy a form of the Golden Rule from the perspective of the social planner: weight the welfare of two heterogeneous agents as you would weight your own welfare if you faced the same variations of endowments and preferences across time. Said plainly, a social planner using Negishi weights puts himself "in the shoes" of the agents that he is planning for.

3. Discussion

This strong ethical result comes with one important caveat – that income endowments in each period for the sole owner are exogenous and that saving and borrowing of income are not feasible. If this were not the case then a forward-looking agent would engage in consumption smoothing through saving and borrowing, allowing numeraire and extractive consumption to flow to periods when it is most highly valued. The correspondence between present discounted utilitarianism for an individual and present discounted Negishi weighted utility remains; however, the tendency of an agent will be to smooth variations in the marginal utility of the numeraire through time. This tendency is analogous to allowing redistributive 'transfers' between past, present and future versions of the agent. Such transfers are seamlessly accomplished with perfect capital markets, and are feasible to varying degrees in real-world capital institutions. The tendency of such transfers would be to smooth the heterogeneity in the Negishi weights for an individual through time. Introducing a market for private capital therefore allows a form of within-agent 'redistribution' that is not permitted at any moment of time in a Negishi-weighted SWF between individuals since Negishi weights respect the endowments of agents at each instant.

Even with private saving or borrowing, Negishi weights retain some ethical appeal from a 'procedural justice' perspective in that the method of weighting within-individual utility and between-individual utility remains consistent. However this appeal is somewhat weakened in the sense that individual consumption-smoothing for an agent with distinct endowments or preferences at different points in time allows freedom of adjustment in the weighting of the 'personal SWF' that is not allowed between individuals in the SWF at a moment of time. While offering a qualification of the strong 'Golden Rule' result, we believe our exposition nevertheless helps to clarify the real source of ethical unease with Negishi weights. Namely, since Negishi weights without private capital accumulation satisfy strong ethical consistency properties, the primary ethical objections may lie more in the ethical asymmetries introduced by private capital markets and initial allocations of resource endowments than with Negishi weighting *per se*.

The above analysis leads to two immediate policy insights for the management of public capital stocks and specifically for climate policy. First, optimal climate policy models that attempt to determine optimal carbon abatement (and hence pricing) for developing vs. non-developing countries in a non-Negishi context (Stanton 2011; Ackerman et al. 2013; Tol 2001) may better address distributional concerns via targeted policies facilitating capital transfers rather than through attempting to address distributive justice issues simultaneously through policies primarily designed to remedy market failures such as un-priced greenhouse gas emissions. Divergent pricing of carbon between 'North' and 'South' may achieve an ethically desirable transfer of endowments, but may do so at substantially greater cost than alternative policies that unbundle climate policy from problems of inequality across nations, such as reductions of trade barriers and capital market reforms.

Second, our analysis provides insight into the controversy concerning appropriate numeraire (money-metric) discount rates for projects involving public capital stocks – particularly for investment problems in which costs and benefits are separated by a wide time horizon, such as climate change. Disagreements about the appropriate numeraire discount rate typically center around two key decisions (P. Dasgupta 2008). The first decision is the choice of the social or utility discount rate – ρ on the RHS of Eq. (2). The second decision is making the correct adjustments to the social discount rate to account for temporal variations in the flows entering the utility function over time and the effects of these flows on the marginal utility of the chosen numeraire. These reflect the additional terms on the RHS of Eq. (2). These adjustments are necessary to quantify the discounted value of lost consumption and to recover optimal carbon or greenhouse gas prices since both are in numeraire (money) units. Typically this is accomplished using a simplified version of the Ramsey discount rate in Eq. (2), focusing purely on the consumption effects of changes in natural capital. This approach ignores the potential for climate to affect welfare directly or through non-market services (Hoel and Sterner 2007) – an effect captured in our model by the inclusion of s in the utility function. In addition to the immediate implications of failing to account for these non-market effects in per-period welfare measures, this simplification also neglects any adjustment of the numeraire discount rate to account for induced changes in the marginal utility of the numeraire as the natural capital stock changes – the fourth RHS term of Eq. (2). The magnitude of the error depends on the rate of degradation of the climate system, $\frac{\dot{s}}{s}$, and the manner in which climate influences the marginal utility of numeraire consumption, η_{zs}. Given the well-known sensitivity of climate policy prescriptions to the discount rate (W. D. Nordhaus 2007; W. Nordhaus 2007), it is possible that changes in the non-market services from climate could have significant impact on the optimal price path for carbon through the numeraire discount rate even if the direct amenity effects of climate change are swamped by climate change's direct effects on consumption.

We have emphasized the use of Negishi market-replicating weights, but the adoption of these normative criteria in policy circles is far from universal, as reflected in actual policy decisions. Sen (2000) notes that there are ample reasons to fault policies dependent on potential compensation arguments as 'improvements' if it is clear that the implied compensation cannot or will not take place. This critique may be particularly salient in cases such as climate change where winners and losers may be separated by wide stretches of time and transcend the jurisdictions of nation-states. In such a world, the explicit incorporation of distributional objectives into the SWF may have a strong ethical justification. Nevertheless, we have demonstrated that the use of non-Negishi weights violates deep-rooted ethical principles familiar to most primary school children – do unto others as you would do unto yourself.

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